College of Science

MMW 101

MATHEMATICS IN THE MODERN WORLD

Ms. Elyssa Grace Antonio

College of Science

Module 5: Elementary Logic

Objectives:

At the end of the module, you should be able to:

- identify propositions,
- form compound propositions, and
- express compound propositions in symbolic forms and vice-versa.

Propositions, Simple and Compound Statements

- A **proposition**, sometimes called a statement, is a declarative sentence that is either true or false, but not both.
- To represent propositions, the lower case letters like p,q,r,s,... are used. These are called propositional variables.

Propositions, Simple and Compound Statements

PROPOSITIONS	NOT PROPOSITIONS
1. Tarlac is a province in Region III. (True)	1. Where is your teacher? (a question)
2. Elephants have wings. (False)	2. Open the door. (a command)
3. $15 - 2x = 8$, if $x = 5$. (False)	3. Ouch! (an exclamation)
4. Twenty is less than fifty. (True)	4. This food is delicious. (subjective)
5. $5^2 + 1$ is a prime number. (False)	5. $5x + 2y = 7$ (There are no assigned values to x and y.)

Propositions, Simple and Compound Statements

- Propositions can either be a simple statement or a compound statement.
- A simple statement is a single statement that does not have other statements as parts.
- A compound statement contains two or more statements joined by logical connectives.

Propositions, Simple and Compound Statements

EXAMPLES

- Two is a rational number.

(simple statement)

- Two is a rational number and four is an even number.

(compound statement)

Propositions, Simple and Compound Statements

The sky is blue.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

The sky is blue.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

Water is liquid at room temperature.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

Water is liquid at room temperature.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

The earth is round and it revolves around the sun.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

The earth is round and it revolves around the sun.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

Dogs are mammals.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

Dogs are mammals.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

If it rains, then the ground will be wet.

COMPOUND

SIMPLE

Propositions, Simple and Compound Statements

If it rains, then the ground will be wet.

COMPOUND

SIMPLE

Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
NEGATION	not	~ curl or tilde ¬ dash with a tail	~ p ¬p	not p
CONJUNCTION	and/but	$\wedge$ caret mark	p $\wedge$ q	p and q
DISJUNCTION	or	$\vee$ wedge or vee	p $\vee$ q	p or q
CONDITIONAL/ IMPLICATION	if..., then	$\rightarrow$ right arrow	p $\rightarrow$ q	if p, then q if p, q p implies q p only if q q if p q whenever p q is necessary for p

Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
BICONDITIONAL	if and only if	$\leftrightarrow$ double-headed arrow	$p \leftrightarrow q$	p if and only if q p implies q, and q implies p

Logical Connectives and Symbols

not

CONJUCTION

DISJUNCTION

NEGATION

CONDITIONAL

BI-CONDITIONAL

Logical Connectives and Symbols

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Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
A. NEGATION	not	~ curl or tilde ¬ dash with a tail	~ p ¬p	not p

Note: The negation does not form a compound statement when used but it is considered a connective since it alters the original statement.

PROPOSITIONS	NEGATION
p: The traffic in EDSA is heavy.	~p: The traffic in EDSA is not heavy.
q: Today is Friday	~q: Today is not Friday.
r: I do not like milk tea.	~r: I like milk tea
s:The students did not attend the webinar	~s: The students attended the webinar.
t: 23 + 34 ≠ 57	~t: 23 + 34 = 57

Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
A. NEGATION	not	~ curl or tilde ¬ dash with a tail	~ p ¬p	not p

Note: The negation does not form a compound statement when used but it is considered a connective since it alters the original statement.

ORIGINAL STATEMENT	NEGATION	ORIGINAL STATEMENT	NEGATION
All A are B	Some A are not B	All dogs are friendly.	Some dogs are not friendly.
Some A are B	No A are B	Some fruits are sweet.	No fruits are sweet.
No A are B	Some A are B	No cars are electric.	Some cars are electric.
Some A are not B	All A are B	Some students are not diligent.	All students are diligent.

Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
B. CONJUNCTION	and/but	$\wedge$ caret mark	$p \wedge q$	p and q

Let p, q, r, and s be the following propositions:

p: Russel is not happy.

q: Russel is going to a party.

r: It is a holiday.

s: Jack is not invited.

Compound Propositions	Symbolic Form
1. Russel is not happy but it is a holiday.	$p \wedge r$
2. It is not a holiday and Russel is happy.	$\sim r \wedge \sim p$
3. Russel is going to a party but Jack is not invited.	$q \wedge s$
4. Jack is invited and Russel is not happy.	$\sim s \wedge p$
5. It is not a holiday and Jack is not invited.	$\sim r \wedge s$

Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
C. DISJUNCTION	or	$\vee$ wedge or vee	$p \vee q$	p or q

Let p, q, r, and s be the following propositions:

p: Jerald is a producer.

q: Jerald is a composer.

r: $2 > 5$

s: $5 < 7$

Compound Propositions	Symbolic Form
1. Jerald is a producer or he is a composer.	$p \vee q$
2. Jerald is not a composer or he is a producer.	$\sim q \vee p$
3. $2 > 5$ or $5 < 7$	$r \vee s$
4. Jerald is not a composer or he is not a producer.	$\sim q \vee \sim p$
5. $2 > 5$ or $5 > 7$	$r \vee \sim s$

Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
D. CONDITIONAL	if..., then	→ right arrow	p → q	if p, then q, if p, q p implies q, p only if q q if p, q whenever p q is necessary for p

In **"if p, then q"**, p is called the antecedent (hypothesis or premise), and q is called the consequent (or the conclusion).

p: It is going to rain. q: I will not go out. r: Cherry is going to visit a friend.

s: All even numbers are integers. t: Zero is an integer.

Compound Propositions	Symbolic Form
1. If it is going to rain, then I will not go out. (in the if p then q form)	p → q
2. I will not go out if it is going to rain. (in the q if p form)	p → q
3. I will not go out whenever it is going to rain. (in the q whenever p form)	p → q
4. If it is not going to rain, then Cherry is going to visit a friend.	~p → r

Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
D. CONDITIONAL	if..., then	→ right arrow	p → q	if p, then q, if p, q p implies q, p only if q q if p, q whenever p q is necessary for p

In **"if p, then q"**, p is called the antecedent (hypothesis or premise), and q is called the consequent (or the conclusion).

p: It is going to rain. q: I will not go out. r: Cherry is going to visit a friend.

s: All even numbers are integers. t: Zero is an integer.

Compound Propositions	Symbolic Form
5. If it is not going to rain, then I will go out.	~p → ~q
6. If all even numbers are integers, then zero is an integer.	s → t
7. If zero is not an integer, then not all even numbers are integers.	~t → ~s
8. If Cherry is not going to visit a friend, then I will go out.	~r → ~q

Logical Connectives and Symbols

PROPOSITIONAL LOGIC	CONNECTIVE	Symbol/ Term	Symbolic Form	Read as
E. BICONDITIONAL	if and only if	↔ double-headed arrow	$p \leftrightarrow q$	p if and only if q p implies q, and q implies p

Let p, q, r, s, and t be the following propositions:

p: Lewis is a singer.

q: Lewis is a vocalist.

r: Lewis is a recording artist.

s: Seven is a whole number.

t: 9 + 13 = 25

Compound Propositions	Symbolic Form
1. Lewis is a vocalist if and only if he is a singer.	$q \leftrightarrow p$
2. Lewis is a recording artist if and only if he is a singer.	$r \leftrightarrow p$
3. Lewis is not a recording artist if and only if he is not a singer.	$\sim r \leftrightarrow \sim p$
4. Seven is not a whole number if and only if 9 + 13 = 25.	$\sim s \leftrightarrow t$
5. 9 + 13 ≠ 25 if and only if seven is not a whole number.	$\sim t \leftrightarrow \sim s$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
r: The bus arrives on time.
s: The exam is difficult.
t: The teacher gives homework.
u: The museum is free on Sundays.

The bus arrives on time, and the museum is not free on Sundays.

1. $\sim r \wedge u$
2. $u \wedge \sim r$
3. $r \vee \sim u$
4. $r \wedge \sim u$
5. $\sim u \vee \sim r$

Logical Connectives and Symbols

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q: The cafeteria serves pasta.
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The bus arrives on time, and the museum is not free on Sundays.

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4. $r \wedge \sim u$
5. $\sim u \vee \sim r$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
r: The bus arrives on time.
s: The exam is difficult.
t: The teacher gives homework.
u: The museum is free on Sundays.

The teacher gives homework, but the library is not open today.

1. $t \wedge \sim p$
2. $p \wedge \sim t$
3. $t \vee \sim p$
4. $\sim t \wedge \sim p$
5. $\sim p \vee \sim t$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
r: The bus arrives on time.
s: The exam is difficult.
t: The teacher gives homework.
u: The museum is free on Sundays.

The teacher gives homework, but the library is not open today.

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2. $p \wedge \sim t$
3. $t \vee \sim p$
4. $\sim t \wedge \sim p$
5. $\sim p \vee \sim t$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
r: The bus arrives on time.
s: The exam is difficult.
t: The teacher gives homework.
u: The museum is free on Sundays.

If the exam is not difficult, then the teacher gives homework.

1. $\sim s \rightarrow \sim t$

2. $s \vee \sim t$

3. $\sim s \leftrightarrow t$

4. $\sim t \leftrightarrow \sim s$

5. $\sim s \rightarrow t$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
r: The bus arrives on time.
s: The exam is difficult.
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u: The museum is free on Sundays.

If the exam is not difficult, then the teacher gives homework.

1. $\sim s \rightarrow \sim t$

2. $s \vee \sim t$

3. $\sim s \leftrightarrow t$

4. $\sim t \leftrightarrow \sim s$

5. $\sim s \rightarrow t$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
r: The bus arrives on time.
s: The exam is difficult.
t: The teacher gives homework.
u: The museum is free on Sundays.

The library is not open today, or the musuem is not free on Sundays.

1. $\sim p \rightarrow \sim u$
2. $p \vee \sim u$
3. $\sim p \vee \sim u$
4. $\sim p \wedge \sim u$
5. $\sim p \rightarrow u$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
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1. $\sim p \rightarrow \sim u$
2. $p \vee \sim u$
3. $\sim p \vee \sim u$
4. $\sim p \wedge \sim u$
5. $\sim p \rightarrow u$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
r: The bus arrives on time.
s: The exam is difficult.
t: The teacher gives homework.
u: The museum is free on Sundays.

The exam is difficult if and only if, the teacher does not give homework

1. $\sim s \rightarrow \sim t$
2. $s \leftrightarrow t$
3. $s \leftrightarrow \sim t$
4. $s \wedge \sim t$
5. $\sim s \leftrightarrow t$

Logical Connectives and Symbols

p: The library is open today.
q: The cafeteria serves pasta.
r: The bus arrives on time.
s: The exam is difficult.
t: The teacher gives homework.
u: The museum is free on Sundays.

The exam is difficult if and only if, the teacher does not give homework

1. $\sim s \rightarrow \sim t$

2. $s \leftrightarrow t$

3. $s \leftrightarrow \sim t$

4. $s \wedge \sim t$

5. $\sim s \leftrightarrow t$

Compound Statements and Grouping Symbols

When a compound statement is to be written in symbolic form, there are instances when simple statements need to be grouped together. This is done by using the parentheses.

Symbolic Form	The parentheses indicate that:
$p \wedge (q \vee r)$	q and r are grouped together
$(p \wedge \sim q) \vee \sim r$	p and ~q are grouped together
$(p \wedge q) \rightarrow (\sim r \wedge \sim s)$	p and q are grouped together. ~r and ~s are grouped together.

Compound Statements and Grouping Symbols

Let p, q, r, and s be the following propositions:

p: Cathy is good in Chemistry.
q: Cathy is good in Statistics.
r: Cathy is a scholar.
s: Cathy is an athlete.

English Sentence	Symbolic Form
1. Cathy is not an athlete, but she is good in Chemistry and she is a scholar.	$\sim s \wedge (p \wedge r)$
2. If Cathy is good in Chemistry and she is good in Statistics, then she is a scholar.	$(p \wedge q) \rightarrow r$
3. Cathy is not good in Chemistry and she is not good in Statistics, but she is a scholar and an athlete.	$(\sim p \wedge \sim q) \wedge (r \wedge s)$

Compound Statements and Grouping Symbols

Let's define propositions:

p: John studies hard.

q: John passes the exam.

r: John is happy.

s: John plays basketball.

John does not play basketball, but he studies hard and is happy.

1. $s \wedge (p \wedge r)$

2. $\sim s \wedge (p \wedge r)$

3. $\sim s \leftrightarrow (p \wedge r)$

4. $\sim s \vee (p \vee r)$

5. $s \rightarrow (p \wedge r)$

Compound Statements and Grouping Symbols

Let's define propositions:

p: John studies hard.

q: John passes the exam.

r: John is happy.

s: John plays basketball.

John does not play basketball, but he studies hard and is happy.

1. $s \wedge (p \wedge r)$

2. $\sim s \wedge (p \wedge r)$

3. $\sim s \leftrightarrow (p \wedge r)$

4. $\sim s \vee (p \vee r)$

5. $s \rightarrow (p \wedge r)$

Compound Statements and Grouping Symbols

Let's define propositions:

p: John studies hard.

q: John passes the exam.

r: John is happy.

s: John plays basketball.

If John studies hard and passes the exam, then he is happy.

1. $(\sim p \vee q) \rightarrow r$
2. $(p \wedge q) \leftrightarrow r$
3. $(p \leftrightarrow q) \rightarrow r$
4. $(\sim p \wedge q) \rightarrow r$
5. $(p \wedge q) \rightarrow r$

Compound Statements and Grouping Symbols

Let's define propositions:

p: John studies hard.

q: John passes the exam.

r: John is happy.

s: John plays basketball.

If John studies hard and passes the exam, then he is happy.

1. $(\sim p \vee q) \rightarrow r$
2. $(p \wedge q) \leftrightarrow r$
3. $(p \leftrightarrow q) \rightarrow r$
4. $(\sim p \wedge q) \rightarrow r$
5. $(p \wedge q) \rightarrow r$

Compound Statements and Grouping Symbols

Let's define propositions:

p: John studies hard.

q: John passes the exam.

r: John is happy.

s: John plays basketball.

John does not study hard and he does not pass the exam, but he is happy and plays basketball.

1. $(\sim p \wedge \sim q) \wedge (r \wedge s)$

2. $(\sim p \wedge q) \wedge (\sim r \wedge s)$

3. $(\sim p \wedge \sim q) \vee (r \wedge s)$

4. $(p \wedge q) \wedge (\sim r \wedge \sim s)$

5. $(\sim p \vee \sim q) \wedge (r \vee s)$

Compound Statements and Grouping Symbols

Let's define propositions:

p: John studies hard.

q: John passes the exam.

r: John is happy.

s: John plays basketball.

John does not study hard and he does not pass the exam, but he is happy and plays basketball.

1. $(\sim p \wedge \sim q) \wedge (r \wedge s)$

2. $(\sim p \wedge q) \wedge (\sim r \wedge s)$

3. $(\sim p \wedge \sim q) \vee (r \wedge s)$

4. $(p \wedge q) \wedge (\sim r \wedge \sim s)$

5. $(\sim p \vee \sim q) \wedge (r \vee s)$

Compound Statements and Grouping Symbols

Let's define propositions:

p: John studies hard.

q: John passes the exam.

r: John is happy.

s: John plays basketball.

If John is happy, then either he studied hard or he passed the exam.

1. $r \rightarrow (\sim p \vee q)$
2. $\sim r \rightarrow (p \vee q)$
3. $r \rightarrow (p \vee q)$
4. $r \rightarrow (\sim p \vee q)$
5. $r \rightarrow (p \wedge q)$

Compound Statements and Grouping Symbols

Let's define propositions:

p: John studies hard.

q: John passes the exam.

r: John is happy.

s: John plays basketball.

If John is happy, then either he studied hard or he passed the exam.

1. $r \rightarrow (\sim p \vee q)$

2. $\sim r \rightarrow (p \vee q)$

3. $r \rightarrow (p \vee q)$

4. $r \rightarrow (\sim p \vee q)$

5. $r \rightarrow (p \wedge q)$